

MPI-PAE/PTh 55/86

December 1986

Fermilab-Pub-87/88-T

## A Consistent Analysis of the $\Delta I = 1/2$ Rule for K Decays

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### ABSTRACT

We calculate the hadronic matrix elements of local operators relevant to the  $K \rightarrow \pi\pi$  decays using the  $1/N$  expansion. This approach permits a consistent treatment of both short and long distance effects. The latter provide important contributions to the enhancement and suppression of the  $\Delta I = 1/2$  and  $\Delta I = 3/2$  amplitudes, respectively. A satisfactory description of the observed data can be achieved within the standard model.

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In the recent papers [1–3] we have analysed the  $K \rightarrow \pi\pi$  decays in the framework of the Standard Model using the  $1/N$  expansion,  $N$  being the number of colours. We have shown that this expansion, while simplifying considerably the usual [4–6], rather complex, short distance analysis provides an effective means for calculating the weak decay amplitudes. Neglecting (small) CP violating effects one finds [1–3] in the large  $N$  limit

$$A(K \rightarrow \pi\pi) = -\frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \sum_{i=1,2,6} z_i(\mu) \langle \pi\pi | Q_i(\mu) | K \rangle \quad (1)$$

where  $z_i$  is the Wilson coefficient function associated with the four quark operator  $Q_i$ ,  $\mu$  is the normalization scale and  $\theta_c$  is the Cabibbo angle. The operators  $Q_i$  are constructed from the light quark fields only ( $u, d, s$ ) and are given as follows

$$\begin{aligned} Q_1 &= (\bar{s}d)_{V-A}(\bar{u}u)_{V-A} & Q_2 &= (\bar{s}u)_{V-A}(\bar{u}d)_{V-A} \\ Q_6 &= -8 (\bar{s}_L q_R)(\bar{q}_R d_L) \end{aligned} \quad (2)$$

where  $(V \pm A)$  refer to  $\gamma_\mu(1 \pm \gamma_5)$ ,  $q_{R(L)} = (1/2)(1 \pm \gamma_5)q$  and sums over colour indices and  $q = (u, d, s)$  in  $Q_6$  are understood. The operators  $Q_1$  and  $Q_2$  are the usual current-current operators, whereas  $Q_6$  is the dominant density-density penguin operator [7]. Since the operators  $Q_i$  in Eq. (2) are constructed from the light quark fields only, the full information about the heavy quark fields ( $c, b, t$ ) is contained in the coefficient functions  $z_i$ . Correspondingly the normalization scale  $\mu$  in Eq. (1) is not completely arbitrary but must be chosen below the charm quark mass. Now as discussed in [2, 3] the physics contributions from scales above  $\mu$  are fully contained in the coefficients  $z_i(\mu)$  whereas the remaining contributions from the low energy physics below  $\mu$  (i.e. from zero momentum to  $\mu$ ) are contained in the matrix elements  $\langle \pi\pi | Q_i(\mu) | K \rangle$ . It follows then that for  $\mu = O(1\text{GeV})$ , the coefficients functions  $z_i(\mu)$  can to a good approximation be calculated within a *quark picture* by means of the usual renormalization group methods. In principle the meson matrix elements can also be computed in a nonperturbative quark-gluon picture where mesons occur as bound states. However it should also be possible to formulate a dual representation of the strong dynamics in terms of hadronic degrees of freedom. In the large  $N$  limit, this representation becomes exact and a full description of the physics can be achieved using a complete set

of interacting meson fields. In this *meson picture* a short distance analysis is exceedingly complex requiring many meson states and complicated interactions. However the long distance analysis is correspondingly simple as only the lightest meson states may be required and the important interactions are largely dictated by the chiral symmetry structure of the meson lagrangian. Hence we will achieve a consistent unified description of the physics by using the quark-gluon picture at short distance matched to the meson picture at long distance with the scale  $\mu$  chosen to minimize the effects of the approximate treatments used in both pictures.

In the quark picture, the scale  $\mu$  enters naturally as the normalization scale in the renormalization group improved perturbative QCD calculations. In the meson picture, the role of  $\mu$  is played by the physical ultraviolet cutoff, to be denoted by  $M$ , which is used to evaluate the meson loop diagrams. If we truncate only on the pseudoscalar mesons, the effective low energy meson theory describing the weak and strong interactions will appear nonrenormalizable and the scale  $M$  will play an essential role. In the evaluation of the matrix elements  $\langle \pi\pi | Q_i(\mu) | K \rangle$  we will set  $\mu = M$ . This identification of  $\mu$  with  $M$  is certainly an idealization in the approximate treatment used below, but can be made precise with a complete description of quark and meson pictures used for the short and long distance physics, respectively.

In what follows we will present and discuss the results obtained for the decay amplitudes  $A(K \rightarrow \pi\pi)$  using a combined treatment of the two physical pictures and a systematic application of  $1/N$  expansion method. The details of the quark picture calculations of  $z_i(\mu)$  have been already presented in [2, 3] where the matrix elements  $\langle \pi\pi | Q_i(M) | K \rangle$  have been given only to leading order in the large  $N$  expansion. The principal novelty of the present letter are the  $1/N$  corrections to  $\langle \pi\pi | Q_i(M) | K \rangle$ , which together with the results of [2, 3] permit a consistent treatment of the amplitudes  $A(K \rightarrow \pi\pi)$  within the large  $N$  approach. The details of the  $1/N$  calculations and the comparison with related approaches [8, 9] are relegated to a separate publication [10].

Our previous analysis [2, 3] combined the  $1/N$  expansion with the usual renormalization group analysis including a proper treatment of the threshold effects in the penguin contributions to give the coefficients  $z_i(\mu)$  listed in Table I for various

values of  $\mu$  and  $\Lambda_{QCD} = 300 \text{ MeV}$ . Roughly the same results for the coefficients  $z_1$  and  $z_2$  are obtained when higher order QCD corrections [11] are included but  $\Lambda_{\overline{MS}} \approx 200 \text{ MeV}$  is used. The corresponding higher order corrections to  $z_6$  are not known.

To calculate the matrix elements  $\langle \pi\pi | Q_i(\mu) | K \rangle$  we will use a truncated chiral Lagrangian describing the low energy strong interactions of pseudoscalar mesons [12, 3]

$$L_{tr} = \frac{f_\pi^2}{4} \left[ \text{tr}(D_\mu U D_\mu U^\dagger) + r \text{tr}(m(U + U^\dagger)) - \frac{r}{\Lambda_\chi^2} \text{tr}(m(D^2 U + D^2 U^\dagger)) \right] \quad (3)$$

where  $U = U(\pi)$  is the unitary chiral matrix describing the octet of pseudoscalar mesons<sup>1)</sup>.  $D_\mu U$  is the usual weak covariant derivative and  $m$  is the real and diagonal quark mass matrix. From the structure of this Lagrangian we can read off the consistent meson representation of the quark currents

$$\bar{q}_L^j \gamma_\mu q_L^i = i \frac{f_\pi^2}{4} \left\{ (\partial_\mu U) U^\dagger - U (\partial_\mu U^\dagger) - \frac{r}{\Lambda_\chi^2} [m(\partial_\mu U^\dagger) - (\partial_\mu U)m] \right\}_{ij} \quad (4)$$

and the quark densities

$$\bar{q}_R^j q_L^i = -\frac{f_\pi^2}{4} r \left[ U - \frac{1}{\Lambda_\chi^2} \partial^2 U \right]_{ij}. \quad (5)$$

The chiral Lagrangian in Eq.(3) must not be viewed as a normal effective tree Lagrangian but instead must be used as a fully interacting field theory including loop effects. In this sense we are providing a bosonization of the fundamental quark theory where all the quark currents and densities have a valid representation in terms of the meson fields. In the truncated version, the meson representation is valid only for a proper description of the long distance physics.

In the leading order the parameter  $r$  can be eliminated in favour of the meson masses

$$m_\pi^2 = \frac{r}{2}(m_u + m_d) \quad m_K^2 \simeq \frac{r}{2}m_s$$

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<sup>1)</sup>The singlet pseudoscalar meson decouples due to a large mass generated by the anomaly.

$$m_8^2 = \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2. \quad (6)$$

The Lagrangian parameter  $f_\pi^2$  determines the coupling strength to lowest order in the meson momentum while  $\Lambda_\chi$  sets the scale of the higher order terms which are always expected in a truncated theory (note that  $\Lambda_\chi$  is a hadronic scale different from  $\Lambda_{QCD}$ ).

The chiral Lagrangian in Eq. (3) contains only terms with a single trace over the flavor indices which reflects the large  $N$  structure of  $QCD$ . The leading  $N$  contributions to any quantity are simply obtained from the tree diagrams whereas the leading  $1/N$  corrections are found by calculating the one-loop contributions. More generally the  $1/N$  expansion corresponds to the loop expansion characterized by the inverse powers of  $f_\pi^2$  ( $f_\pi^2 \sim N$ ) with the strong interaction vertices given by the truncated Lagrangian of Eq. (3).

There are some similarities between our calculations and the usual chiral perturbative calculations found in the literature [13–15]. On the other hand our approach distinguishes itself in two important ways. First, the large  $N$  structure of the basic truncated low energy Lagrangian of Eq.(3) provides a simplification over those effective Lagrangians usually considered. More importantly our loop calculations employ a cut-off regularization and consequently our results exhibit a *quadratic* dependence on the physical cut-off  $M$ . This quadratic dependence is lost in the usual chiral perturbative calculations [13–15] which are based on the dimensional regularization. In effect, dimensional regularization makes extra infrared subtractions of quadratically divergent terms. These subtractions are not permitted in the full integration of the loop contributions to the truncated theory. As we will see below the quadratic dependence on the physical cut-off is an essential ingredient in the matching of the meson and quark pictures. This quadratic dependence is fully consistent with chiral symmetry and also stabilizes the  $1/N$  expansion.

The evolution of the coefficient functions  $z_i(\mu)$  of Eq.(1) is of order  $1/N$  [2]. To consistently evaluate the weak decay amplitudes we must also compute the matrix elements  $\langle \pi\pi | Q_i | K \rangle$  to the same order in  $1/N$ . Hence we must include both the tree and one loop contributions to the matrix elements.

In order to determine the Lagrangian parameters  $f_\pi$  and  $\Lambda_\chi$  we evaluate the

diagrams of Fig. 1. In the  $SU(2)$  limit we obtain the physical pion and kaon decay constants

$$F_\pi = f_\pi \left[ 1 + \frac{m_\pi^2}{\Lambda_\chi^2} - \frac{1}{2} \frac{1}{f_\pi^2} (2 I_2(m_\pi^2) + I_2(m_K^2)) \right] \quad (7)$$

$$F_K = f_\pi \left[ 1 + \frac{m_K^2}{\Lambda_\chi^2} - \frac{3}{8} \frac{1}{f_\pi^2} (2 I_2(m_K^2) + I_2(m_\pi^2) + I_2(m_8^2)) \right] \quad (8)$$

and the ratio

$$\frac{F_K}{F_\pi} = 1 + \frac{m_K^2 - m_\pi^2}{\Lambda_\chi^2} - \frac{1}{8} \frac{1}{f_\pi^2} [2 I_2(m_K^2) + 3 I_2(m_8^2) - 5 I_2(m_\pi^2)] \quad (9)$$

where

$$I_2(m_i^2) = \frac{1}{(4\pi)^2} [M^2 - m_i^2 \ln(1 + \frac{M^2}{m_i^2})] \quad (10)$$

with  $M$  denoting the cut-off of the truncated meson theory.

Eqs. (7-9) are the first non-trivial generalizations of Eqs. (2.11)-(2.13) of ref. [3] beyond the leading order of  $1/N$  expansion. We note that the quadratic dependence of the cut-off  $M$  cancels in the ratio in Eq. (9).

Neglecting the  $m_\pi^2/\Lambda_\chi^2$  term in Eq. (7) the effective inverse coupling strength  $f_\pi^2$  at scale  $M$  is given by

$$f_\pi^2 = f_\pi^2(M^2) = F_\pi^2 + 2 I_2(m_\pi^2) + I_2(m_K^2). \quad (11)$$

Using  $F_\pi = 93 \text{ MeV}$  we find  $f_\pi = 116, 124$  and  $134 \text{ MeV}$  for  $M = 0.6, 0.7$  and  $0.8 \text{ GeV}$  respectively. It is amusing to note that the quadratic dependence on the cutoff scale implies a kind of asymptotic freedom behaviour of the running coupling constant.

Using the experimental value  $F_K/F_\pi = 1.28$ , we extract from Eq.(9) the value of  $\Lambda_\chi$  which is essentially independent of  $M$  (it changes by less than 2% in the range of  $M$  considered)

$$\Lambda_\chi \simeq 1000 \text{ MeV} \quad (12)$$

to be compared with the leading order result [3],  $\Lambda_\chi \simeq 900 \text{ MeV}$ . Our result should be contrasted with the one of ref. [12]. Although the formulae for  $F_K/F_\pi$  in ref.

[12] and in the present paper agree,<sup>2)</sup> the authors of [12] did not take into account the cut-off dependence of  $f_\pi$ . Their treatment would give  $\Lambda_\chi = 1000, 1100, 1230$  and  $1580 \text{ MeV}$  for  $M = 0.6, 0.7, 0.8$  and  $1.0 \text{ GeV}$  respectively, i.e. a strong cut-off dependence of  $\Lambda_\chi$  and a large next to leading order correction to its leading order value, which could cast doubt on the validity of the  $1/N$  expansion. However, as we have shown above the proper inclusion of the cut-off dependence in  $f_\pi$  stabilizes the  $1/N$  expansion and gives a roughly cut-off independent  $\Lambda_\chi$ .

With these results in hand, we can now evaluate the matrix elements  $\langle \pi\pi | Q_i(M) | K \rangle$ . We begin with the operators  $Q_1$  and  $Q_2$ . The leading order contributions (tree diagrams) to their matrix elements have been already calculated in [1–3] and we will concentrate here on the  $1/N$  corrections. There are four kinds of one loop diagrams shown in Fig. 2. The diagram in Fig. 2d is a tadpole diagram which must be included for the consistency of our approach. The diagrams in Fig. 2a contain only a five-meson weak vertex whereas the remaining diagrams have in addition to a three-meson weak vertex also a four-meson strong vertex. For a given operator the relevant weak vertices can be obtained by inserting the meson representation of quark currents (see Eq. (4)) into Eq. (2) and expanding the result in powers of  $1/f_\pi^2$ . The strong vertices are inferred from the expansion of  $L_{tr}$  in powers of  $1/f_\pi^2$ . The diagrams in which both internal lines originate from the same quark current are *factorizable*. The remaining diagrams are *non-factorizable*. The full calculation, although straightforward, is rather tedious and will be discussed in detail in [10]. The final result reduces however to simple expressions if  $SU(2)$  symmetry is assumed and Eqs. (7)–(9) are used.

We find

$$X_1 \equiv \langle \pi^+ \pi^- | Q_1(M^2) | K^0 \rangle = \frac{1}{f_\pi^2} X F_1(M^2) \quad (13)$$

$$X_2 \equiv \langle \pi^+ \pi^- | Q_2(M^2) | K^0 \rangle = X_F + \frac{1}{f_\pi^2} X F_2(M^2) \quad (14)$$

$$X_3 \equiv \langle \pi^0 \pi^0 | Q_1(M^2) | K^0 \rangle = -X_F - \frac{1}{f_\pi^2} X F_3(M^2) \quad (15)$$

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<sup>2)</sup>The logarithmic behaviour in Eqs. (7) and (8) agrees also with the one found in ref. [13].

$$X_4 \equiv \langle \pi^0 \pi^0 | Q_2(M^2) | K^0 \rangle = \frac{1}{f_\pi^2} X [F_2(M^2) - F_1(M^2) - F_3(M^2)] \quad (16)$$

$$X_5 \equiv \langle \pi^+ \pi^0 | Q_{1,2}(M^2) | K^+ \rangle = \frac{1}{\sqrt{2}} [X_2 - X_4] = \frac{1}{\sqrt{2}} [X_1 - X_3] \quad (17)$$

where

$$X_F = X \left[ 1 + \frac{f_\pi}{F_\pi} \left( \frac{F_K}{F_\pi} - 1 \right) \frac{m_\pi^2}{m_K^2 - m_\pi^2} \right] \quad (18)$$

is the factorizable contribution and

$$X = \sqrt{2} F_\pi (m_K^2 - m_\pi^2). \quad (19)$$

The cut-off dependent functions  $F_1$ ,  $F_2$  and  $F_3$  are given as follows:

$$F_1(M^2) = \frac{1}{(4\pi)^2} \left[ \frac{f_\pi}{F_\pi} \right] \left[ -2 M^2 + \left( \frac{m_K^2}{4} + \frac{19}{9} m_\pi^2 \right) \ln \left( 1 + \frac{M^2}{\tilde{m}^2} \right) \right] \quad (20)$$

$$F_2(M^2) = \frac{1}{(4\pi)^2} \left[ \frac{f_\pi}{F_\pi} \right] \left[ M^2 + \left( m_K^2 - \frac{3}{2} m_\pi^2 \right) \ln \left( 1 + \frac{M^2}{\tilde{m}^2} \right) \right] \quad (21)$$

and

$$F_3(M^2) = \frac{1}{(4\pi)^2} \left[ \frac{f_\pi}{F_\pi} \right] \frac{8}{9} m_\pi^2 \ln \left( 1 + \frac{M^2}{\tilde{m}^2} \right). \quad (22)$$

The ratio  $f_\pi/F_\pi$  in Eqs. (18) and (20)-(22) appears because  $f_\pi^{-2}$  is the expansion parameter. The arguments of the logarithmic terms are only approximations since the mass scale  $\tilde{m}$  replaces a rather complicated dependence of the exact expressions on the meson masses. It turns out, however, that our results are not very sensitive to the value of  $\tilde{m}$ ,  $m_\pi < \tilde{m} < m_K$ . The values of  $X_i$  are given for different values of  $M$  in Table I.

The following remarks are in order:

i)  $X_F$  in Eq.(18) represents the factorizable contributions and the remaining terms in Eqs.(13)-(17) come from non-factorizable diagrams (they are all non-leading).

ii) we observe that all the *factorizable* contributions to  $\langle \pi\pi | Q_i | K \rangle$  can be expressed in terms of the physical decay constants  $F_\pi$  and  $F_K$ . Note that the factorizable contributions do not include the usual Fierz rearrangements of the quark operators. Due to the effective bosonization, Fierz terms will appear



as a part of the non-factorizable loop corrections. We conclude therefore that although the vacuum insertion method gives correct results for the leading in  $N$  contributions [1] it completely misrepresents the next to leading effects.

iii) we note that the next to leading order corrections to the hadronic matrix elements vanish for  $M = 0$  implying that the leading order in  $1/N$  expansion corresponds to the zero momentum limit. Thus the inclusion of the next to leading order corrections can be viewed as taking into account the physics contributions from the momentum range from 0 to  $M$ . These new contributions make  $X_1$  and  $X_4$  non-zero and increase  $X_2$  over its leading order ( $M = 0$ ) value. Since the increase of  $X_4$  is stronger than that of  $X_2$  the matrix element  $X_5$  which governs the  $\Delta I = 3/2$  amplitudes is further suppressed. This pattern is not only welcomed by the data but, in addition, being the same pattern as in the quark picture, allows a plausible matching of the meson and quark evolutions. In fact this matching is necessary for a consistent analysis of the  $K$  decay amplitudes.

iv) in the limit  $m_\pi \rightarrow 0$  Eqs. (13)-(17) can be cast into the following operator relations:

$$Q_1(M^2) = Q_1(0) + \frac{1}{f_\pi^2} F_1(M^2) Q_2(0) \quad (23)$$

$$Q_2(M^2) = Q_2(0) + \frac{1}{f_\pi^2} F_1(M^2) Q_1(0) + \frac{1}{f_\pi^2} F_2(M^2)[Q_2(0) - Q_1(0)] \quad (24)$$

which show that the inclusion of the next to leading order corrections to hadronic matrix elements can indeed be viewed as the evolution of the operators (*the meson evolution*) from zero momentum to  $M$ .

In order to complete the analysis of the amplitudes  $A(K \rightarrow \pi\pi)$  we still need the matrix elements of the operator  $Q_6$ . Since the coefficient  $z_6$  is  $O(1/N)$ , it suffices to use the leading order result [3, 12]

$$\langle \pi^+ \pi^- | Q_6 | K^0 \rangle = \langle \pi^0 \pi^0 | Q_6 | K^0 \rangle = Y \quad (25)$$

with

$$Y = -4\sqrt{2}f_\pi \left[ \frac{m_K^2}{m_s} \right]^2 \frac{(m_K^2 - m_\pi^2)}{\Lambda_\chi^2} \quad (26)$$

and  $\langle \pi^+ \pi^0 | Q_6 | K^+ \rangle$  being zero. Here  $m_s$  stands for the running strange quark mass,  $m_s(\mu)$ , whose  $\mu$  dependence in the large  $N$  limit is given as follows

$$m_s^2(\mu) = m_s^2(1 \text{ GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(1 \text{ GeV})} \right]^{9/11}. \quad (27)$$

The  $\mu$  dependence of  $1/m_s^2(\mu)$  represents through Eq. (26) the diagonal evolution of  $Q_6$  (i.e. the evolution in the absence of the mixing with the operator  $Q_2$ ) and cancels exactly the corresponding  $\mu$  dependence of  $z_6$ . The remaining  $\mu$  dependence of  $z_6$  is a consequence of the mixing of the operators  $Q_6$  and  $Q_2$  and should be cancelled by a part of the  $M$  dependence of the operator  $Q_2$  given in Eq. (24). We will verify the degree of this cancellation below. The values of  $Y$  for different values of  $M = \mu$  and  $m_s(1 \text{ GeV}) = 125 \text{ MeV}$  are given in Table I. They should be compared with  $Y = -0.58 \text{ GeV}^3$  obtained in [2, 3] where the  $\mu$  dependence of  $m_s$  has been neglected and  $\Lambda_\chi = 0.9 \text{ GeV}$  and  $f_\pi = F_\pi = 93 \text{ MeV}$  have been used.

In Table II we show the values of the amplitudes  $T_1 \equiv A(K^0 \rightarrow \pi^+ \pi^-)$ ,  $T_2 \equiv A(K^0 \rightarrow \pi^0 \pi^0)$  and  $T_3 \equiv A(K^+ \rightarrow \pi^+ \pi^0)$  for  $\Lambda_{QCD} = 300 \text{ MeV}$  and various  $\mu = M$  as obtained using the matrix elements of the present letter (Full) (see Table I) and the leading order matrix elements (L.O.) of ref. [3]. Furthermore we have used  $\tilde{m} = 0.3 \text{ GeV}$ . The following observations can be made on the basis of this Table:

i) The amplitudes  $T_1$  and  $T_2$  obtained using the matrix elements of Eqs. (13)-(17) are almost  $\mu$  independent implying that the matching of the quark and meson pictures is plausible. This weak  $\mu$  dependence should be contrasted with the strong  $\mu$  dependence of the amplitudes (L.O.) in which only the leading order matrix elements have been used. Comparison of these two cases also shows that the evolution in the quark picture is continued with the same pattern in the meson picture.

ii) In the case of the  $\Delta I = 3/2$  amplitude  $T_3$  there is a visible  $\mu$  dependence left over. One should however realize that  $T_3$  is a small amplitude and it is probably not surprising that our truncation of the meson physics produces a residual  $\mu$  dependence.

iii) Our results give a satisfactory description of the data within our approximate treatment of the quark and meson contributions. The experimentally ob-

served  $\Delta I = 1/2$  rule is clearly exhibited and the obtained value of the  $\Delta I = 3/2$  amplitude is consistent with the data. Taking  $T_3 = 1.77 \cdot 10^{-8} \text{ GeV}$  and varying  $m_s(1 \text{ GeV}) \equiv m_s$ , we find for  $\tilde{m} = 0.3 \text{ GeV}$

$$\frac{A(K^0 \rightarrow \pi^+\pi^-)}{A(K^+ \rightarrow \pi^+\pi^0)} = \frac{T_1}{T_3} = \begin{cases} 12.9 & m_s = 125 \text{ MeV} \\ 11.6 & m_s = 150 \text{ MeV} \\ 10.7 & m_s = 175 \text{ MeV} \end{cases} \quad (28)$$

to be compared with  $(T_1/T_3)_{exp.} = 15.0$ . Although small values of  $m_s$  are preferred we observe that even for  $m_s = 175 \text{ MeV}$  a large fraction of the  $\Delta I = 1/2$  amplitude is reproduced. Our final results are not very sensitive to the value of  $m_s$ , which indicates that the Penguin contributions to the amplitudes  $T_1$  and  $T_2$  are not dominant. Indeed their contributions to the ratio in Eq.(28) are 4.3, 3.0 and 2.1 for the three values of  $m_s$  considered. Therefore the main bulk of the  $\Delta I = 1/2$  rule comes from the usual octet enhancement [16] which is amplified by the long distance contributions considered here.

iv) We have investigated the sensitivity of our results to the value of  $\tilde{m}$ . For  $\tilde{m}$  in the range  $m_K > \tilde{m} > m_\pi$ ,  $M = \mu = 0.7 \text{ GeV}$  and  $m_s = 125 \text{ MeV}$  we have found in units of  $10^{-8} \text{ GeV}$   $21.7 < T_1 < 23.5$ ,  $19.6 < T_2 < 20.8$  and  $1.45 < T_3 < 1.87$ . Similar results are obtained for other values of  $M$ . One observes that the amplitudes  $T_1$  and  $T_2$  only weakly depend on  $\tilde{m}$  whereas a slightly stronger dependence is observed for  $T_3$ .

The details of our analysis, together with the comparison with other approaches and the discussion of other possible contributions not included here will be discussed in ref. [10].

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$\mu[GeV]$	0.6	0.7	0.8	<i>L.O.</i>
$z_1$	-1.00	-0.87	-0.79	
$z_2$	1.61	1.51	1.44	
$z_6$	-0.110	-0.082	-0.065	
$X_1$	-0.010	-0.013	-0.016	0.0
$X_2$	0.044	0.047	0.049	0.031
$X_3$	-0.032	-0.032	-0.032	-0.031
$X_4$	0.022	0.028	0.033	0.0
$X_5$	0.016	0.013	0.011	0.022
$Y$	-0.38	-0.47	-0.56	-0.58

Table I. The coefficient functions  $z_i$  and the matrix elements  $X_i$  in units of  $GeV^3$  for different values of  $\mu = M$ . In evaluating  $X_i$ , the scale  $\tilde{m} = 0.3 GeV$  has been used. The last column gives the leading order matrix elements of ref. [2](L.O.). The values for  $z_i$  correspond to  $\Lambda_{QCD} = 0.3 GeV$ . The values of  $Y$  correspond to  $m_s(1 GeV) = 125 MeV$ .

$m_s[MeV]$	$\mu[GeV]$	Type	$T_1$	$T_2$	$T_3$
125	0.6	Full	22.8	20.3	1.77
		L.O.	21.2	17.6	2.49
125	0.7	Full	22.3	20.1	1.60
		L.O.	17.6	13.9	2.61
125	0.8	Full	22.2	20.3	1.35
		L.O.	15.3	11.6	2.65
150	0.6	Full	20.5	18.0	1.77
		L.O.	15.3	11.8	2.49
150	0.7	Full	20.2	17.9	1.60
		L.O.	14.9	11.2	2.61
150	0.8	Full	20.2	18.3	1.35
		L.O.	13.2	9.4	2.65
175	0.6	Full	19.0	16.5	1.77
		L.O.	17.5	14.0	2.49
175	0.7	Full	18.8	16.6	1.60
		L.O.	13.2	9.5	2.61
175	0.8	Full	18.9	17.0	1.35
		L.O.	11.9	8.1	2.65
Data			27.7	26.3	1.84

Table II. The amplitudes  $A(K \rightarrow \pi\pi)$  in units of  $10^{-8} GeV$  for different values of  $m_s(1 GeV)$  and  $\mu$  as obtained using the matrix elements of the present letter (Full) and the leading order matrix elements of ref. [2](L.O.).  $\Lambda_{QCD} = 0.3 GeV$  has been used.

### Figure captions

Fig.1 The one loop diagrams contributing to a) wave function renormalization and b) the meson decay constants. The solid circle is the strong vertex from  $L_{tr}$  of Eq. (3). The solid internal lines represent the propagators of the pseudo-scalar meson octet.

Fig.2 The one loop diagrams contributing to the matrix elements  $\langle \pi\pi | Q_i | K \rangle (i = 1, 2)$ . The solid square is a weak operator vertex from Eq. (2) with the quark currents given in Eq. (4). The solid circle is the strong vertex from  $L_{tr}$ .

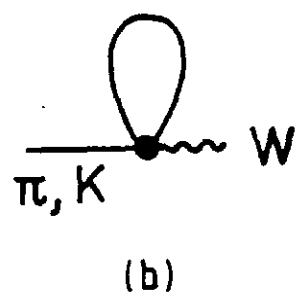
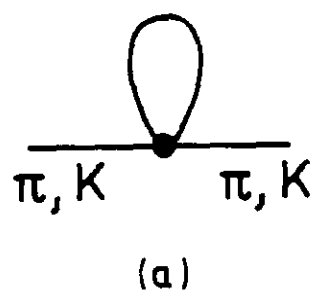


Fig. 1



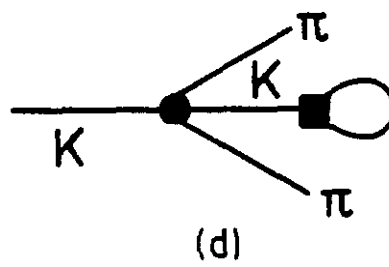
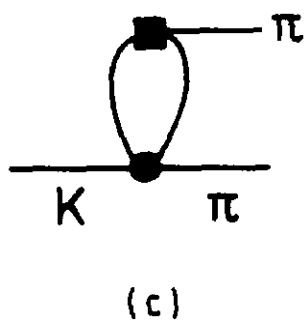
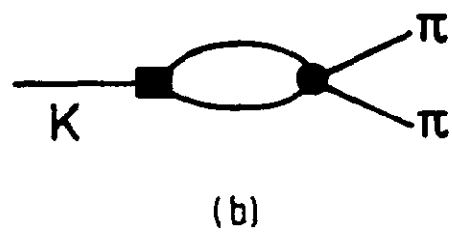
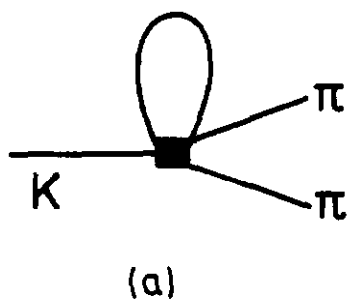


Fig. 2